Statistical Coalescence Model with Exact Charm Conservation

M.I. Gorenstein[a,b,1], **A.P. Kostyuk**[a,b,2], **H. Stöcker**[a,3] and **W. Greiner**[a,4]

[a] Institut für Theoretische Physik, Universität Frankfurt, Germany
[b] Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine

Abstract

The statistical coalescence model for the production of open and hidden charm is considered within the canonical ensemble formulation. The data for the J/ψ multiplicity in Pb+Pb collisions at 158 A·GeV are used for the model prediction of the open charm yield which has not yet been measured in these reactions.

The charmonium states J/ψ and ψ' have been measured in nucleus-nucleus (A+A) collisions at CERN SPS over the last 15 years by the NA38 and NA50 Collaborations. This experimental program was motivated by a suggestion [1] to use the J/ψ as a probe of the state of matter created in the early stage of the collision. In this approach a significant suppression of J/ψ production relative to Drell–Yan lepton pairs is predicted when going from peripheral to central Pb+Pb interactions at 158 A·GeV. This is originally attributed to the formation of a quark-gluon plasma, but could be also explained in microscopic hadron models as secondary collision effects (see [2] and references therein).

The statistical approach, formulated in Ref.[3], assumes that J/ψ mesons are created at hadronization according to the available hadronic phase-space. In this model the J/ψ yield is **independent** of the open charm yield. The model offers a natural explanation of the proportionality of the J/ψ and pion yields and the magnitude of the J/ψ multiplicity in hadronic and nuclear collisions.

Recently the statistical coalescence model was introduced for the charmonium production in Ref.[4]. Similar to the statistical model [3], the charmonium states are assumed to be formed at the hadronization stage. However, they are produced as a coalescence of created earlier c-$\bar{c}$ quarks and therefore the multiplicities of open and hidden charm hadrons are **connected** in that model [4]. The numbers of c-$\bar{c}$ quarks are restricted to the values expected within the pQCD approach. It seemed to be larger than the equilibrium hadron gas (HG) result. This requires the introduction of a new parameter in the HG approach [4] – the charm enhancement factor g_c. This is analogous to the introduction of strangeness suppression factor γ_s [5] in the HG model, where the total strangeness

[1]E–mail: goren@th.physik.uni-frankfurt.de
[2]E–mail: kostyuk@th.physik.uni-frankfurt.de
[3]E–mail: stoecker@th.physik.uni-frankfurt.de
[4]E–mail: greiner@th.physik.uni-frankfurt.de

observed is smaller than its thermal equilibrium value. Within this approach the open charm hadron yield is enhanced by a factor g_c and charmonium yield by a factor g_c^2 in comparison with the equilibrium HG predictions. The enhancement factor is found to be equal to $g_c \cong 1.38$ [4]. How can both Ref.[3] and [4] be compatible to the $\langle J/\psi \rangle$ data? This is because of larger hadronization temperature $T \cong 175$ MeV assumed in [3]. As the thermal equilibrium value of the J/ψ multiplicity increases strongly with T, this larger value of temperature leads to the increase of $\langle J/\psi \rangle$ by a factor of 2, similar to the enhancement $g_c^2 \cong 2$ of the J/ψ multiplicity in Ref.[4].

The thermal HG calculations in Ref.[4] are done within the grand canonical ensemble (g.c.e.) formulation. The validity of the g.c.e. results for the charm hadron yield was questioned in Ref.[4]. As the total number of charm hadrons is expected to be smaller than unity even for the most central Pb+Pb collisions an exact charm conservation within the canonical ensemble (c.e.) should be imposed[5]. In this letter stimulated by the above proposal we consider the c.e. HG formulation for the physical picture suggested in Ref.[4]. The experimental value of the J/ψ multiplicity $\langle J/\psi \rangle$ will be then used to predict the open charm yield within a statistical coalescence model.

The main assumption of Ref.[4] is formulated as

$$N_{c\overline{c}}^{dir} ~=~ \frac{1}{2}\, g_c\, N_O + ~g_c^2~ N_H ~, \tag{1}$$

where N_O is the total thermal multiplicity of all open charm and anticharm mesons and (anti)baryons and N_H is the total thermal multiplicity of particles with hidden charm. Note that open charm resonance states (not included in [4]) give essential contribution[6] to N_O. The number of directly produced $c\overline{c}$ pairs $N_{c\overline{c}}^{dir}$ in the hard collisions is estimated in Ref.[4] to be equal to $N_{c\overline{c}}^{dir} \cong 0.17$ for Pb+Pb SPS collisions with $N_p = 400$ participants. This number is however not quite confident. Recent analysis of dimuon spectrum measured in central Pb+Pb collisions at 158 A$\cdot$GeV by NA50 Collaboration [6] suggests a significant enhancement of dilepton production in the intermediate mass region (1.5$\div$2.5 GeV) over the standard sources. The primary interpretation attributes this observation to the enhanced production of open charm [6]: about 3 times above the pQCD prediction for the open charm yield in Pb+Pb collisions at SPS.

In Ref.[4] N_O and N_H are calculated in the g.c.e.. In the g.c.e. the thermal multiplicities of both open charm and charmonium states are given as (Bose effects are negligible):

$$N_j ~=~ \frac{d_j V~e^{\mu_j/T}}{2\pi^2}~T~m_j^2~K_2\left(\frac{m_j}{T}\right) ~\cong~ d_j~V~e^{\mu_j/T}~\left(\frac{m_jT}{2\pi}\right)^{3/2}~\exp\left(-\frac{m_j}{T}\right)~, \tag{2}$$

when V and T correspond to the volume[7] and temperature of HG system, m_j, d_j denote particle masses and degeneracy factors and K_2 is the modified Bessel function. The particle chemical potential μ_j in Eq.(2) is defined as

$$\mu_j ~=~ b_j\mu_B ~+~ s_j\mu_S ~+~ c_j\mu_C ~, \tag{3}$$

[5]This was first suggested by K. Redlich and B. Müller (quoted in Ref.[4] and L. McLerran, private communication).

[6]We are thankful to Braun-Munzinger and Stachel for pointing this out.

[7]To avoid further complications we use ideal HG formulae and neglect excluded volume corrections.

where b_j, s_j, c_j denote the baryonic number strangeness and charm of particle j. The baryonic chemical potential μ_B regulates the baryonic density of the HG system whereas strange μ_S and charm μ_C chemical potentials should be found from the requirement of zero value for the total strangeness and charm in the system (in our consideration we neglect small effects of the non-zero electrical chemical potential).

In the c.e. formulation (i.e. when the requirement of zero "charm charge" of the HG is used in the exact form) the thermal charmonium multiplicities are still given by Eq.(2) as charmonium states have zero charm charge. The multiplicities (2) of open charm hadrons will however be multiplied by an additional 'canonical suppression' factor (see e.g. [9]). This suppression factor is the same for all individual open charm states. It leads to the total open charm multiplicity N_O^{ce} in the c.e.:

$$N_O^{ce} \;=\; N_O \,\frac{I_1(N_O)}{I_0(N_O)}\;, \tag{4}$$

where N_O is the total g.c.e. multiplicity of all open charm and anticharm mesons and (anti)baryons calculated with Eq.(2) and I_0, I_1 are the modified Bessel functions. For large open charm multiplicity $N_O >> 1$ one finds $I_1(N_D)/I_0(N_O) \rightarrow 1$ and therefore $N_O^{ce} \rightarrow N_O$, i.e. the g.c.e. and c.e. results coincide. For $N_O << 1$ one has $I_1(N_O)/I_0(N_O) \cong N_O/2$ and $N_O^{ce} \cong N_O \cdot N_O/2$, therefore, N_O^{ce} is strongly suppressed in comparison to the g.c.e. result N_O .

Assuming the presence of the charm enhancement factor g_c the coalescence model within the c.e. should now be formulated as:

$$N_{c\bar{c}}^{dir} \;=\; \frac{1}{2}\, g_c\, N_O\, \frac{I_1(g_c N_O)}{I_0(g_c N_O)} \;+\; g_c^2\, N_H\;. \tag{5}$$

The logic of Ref.[4] is the following: 1) input $N_{c\bar{c}}^{dir}$ number (it is assumed to be equal to 0.17 for $N_p = 400$) into Eq.(1); 2) calculate the g_c value; 3) obtain J/ψ multiplicity as $\langle J/\psi\rangle = g_c^2 N_{J/\psi}$, where $N_{J/\psi}$ is given by Eq.(2). Note that the second term in both Eq.(1) and (5) gives only a tiny correction to the first term. Therefore, $g_c \cong 2N_{c\bar{c}}^{dir}/N_O$.

Our consideration differs from Ref.[4] in three points. First, we use Eq.(5) instead of (1). Second, in our calculations we take into account all known particles and resonances with open and hidden charm [10]. Third, we will proceed with Eq.(5) in the reverse way. As the $\langle J/\psi\rangle$ multiplicities have been measured in Pb+Pb collisions at 158 A·GeV for different values of N_p, we start from the requirement:

$$\langle J/\psi\rangle \;=\; g_c^2\; N_{J/\psi}^{tot}\;, \tag{6}$$

to fix the g_c factor. In Eq.(5) the J/ψ thermal multiplicity is calculated as

$$N_{J/\psi}^{tot} = N_{J/\psi} \;+\; R(\psi')N_{\psi'} \;+\; R(\chi_1)N_{\chi_1} \;+\; R(\chi_2)N_{\chi_2}\;, \tag{7}$$

where $N_{J/\psi}$, $N_{\psi'}$, N_{χ_1}, N_{χ_2} are given by Eq.(2) and $R(\psi') \cong 0.54$, $R(\chi_1) \cong 0.27$, $R(\chi_2) \cong 0.14$ are the decay branching ratios of the excited charmonium states into J/ψ. Eq.(5) will be used then to calculate the value of $N_{c\bar{c}}^{dir}$. This value will considered as a prediction of the statistical coalescence model: the open charm yield has not yet been measured in Pb+Pb collisions at SPS.

We use two different sets of the chemical freeze-out parameters:

A: $T = 168$ MeV, $\mu_B = 266$ MeV, $\gamma_s = 1$ Ref.[7];

B: $T = 175$ MeV, $\mu_B = 240$ MeV, $\gamma_s = 0.9$ Ref.[8].

They both were fixed by the HG model fit to the hadron yields data in Pb+Pb collisions at 158 A·GeV (the inclusion of open charm and charmonium states does not modify the rest of the hadron yields). For the fixed number of participants $N_p = 400$ the volume V is defined then from $N_p = V n_B$, where $n_B = n_B(T, \mu_B, \gamma_s)$ is the baryonic density calculated in the g.c.e.. With two sets of the chemical freeze-out parameters **A** and **B** we find $N^{tot}_{J/\psi}$ and N_O values using Eq.(2), calculate g_c factors from Eq.(6) and then calculate $N^{dir}_{c\bar{c}}$ from Eq.(5). Results are the following:

$$A: \quad N^{tot}_{J/\psi} \cong 2.2 \cdot 10^{-4}\ , \quad N_O \cong 1.05\ , \quad g_c \cong 1.3\ , \quad N^{dir}_{c\bar{c}} \cong 0.40 \tag{8}$$

$$B: \quad N^{tot}_{J/\psi} \cong 4.4 \cdot 10^{-4}\ , \quad N_O \cong 1.55\ , \quad g_c \cong 0.95\ , \quad N^{dir}_{c\bar{c}} \cong 0.44 \tag{9}$$

In the above c.e. consideration with exact charm conservation the g_c parameter regulates the **average** number $N_{c\bar{c}}$ of $(c\bar{c})$-pairs in the HG. Therefore, $N_c = N_{\bar{c}}$ is restricted exactly (the c.e.), but the value of $N_c + N_{\bar{c}}$ is restricted in the average sense (the g.c.e). The above c.e. calculations are based on the statistical model distribution for for probabilities to observe $0, 1, 2, ...$ $(c\bar{c})$-pairs in the equilibrium HG. One needs then an additional parameter g_c to adjust these statistical probabilities to the required number of $N^{dir}_{c\bar{c}}$. Another way is to restrict also the $N_c + N_{\bar{c}}$ numbers in the c.e. calculations and use non-statistical probabilities to create $N_{c\bar{c}} = 0, 1, 2, ...$ of $c\bar{c}$-pairs in hard collisions. Because of the assumed hard scattering origin of the $c\bar{c}$ production the Poisson distribution $P(k) = f^k \exp(-f)/k!$ looks quite natural ($k = 0, 1, 2, ...$ is the number of pairs created, $f = N^{dir}_{c\bar{c}}$ is the average number of pairs). The calculations with these 'dynamical' probabilities contain no additional free parameter. All 'dynamical' information needed for the c.e. calculation is now given by the value of f (g_c does not appear). The result for J/ψ yield in this case is:

$$\langle J/\psi \rangle \ = \ f(f+1)\, \frac{N^{tot}_{J/\psi}}{(N_O/2)^2 + N_H}\ , \tag{10}$$

where N_O, N_H and $N^{tot}_{J/\psi}$ multiplicities are calculated in the g.c.e. with Eq.(2). Using the experimental values for $\langle J/\psi \rangle$ one obtains from Eq.(10) the average number of $c\bar{c}$-pairs $f = N^{dir}_{c\bar{c}}$. The results (see Table **A** and **B**) appear to be rather close to those obtain with statistical probabilities. The reason of this fact is that states with $k = 0$ and $k = 1$ dominate in both statistical and 'dynamical' probability distributions. To illustrate this lets consider an extreme choice: the HG states with $N_c = N_{\bar{c}} = 1$ appear with probability $f = N^{dir}_{c\bar{c}}$, the HG states with $N_c = N_{\bar{c}} = 0$ appear with a probability $1 - f$ and states with more than one $(c\bar{c})$-pairs are neglected. Under these restrictions the J/ψ multiplicity becomes equal to:

$$\langle J/\psi \rangle \ = \ f\, \frac{N^{tot}_{J/\psi}}{(N_O/2)^2 + N_H}\ . \tag{11}$$

One sees that Eq.(11) is close to Eq.(10) if $f << 1$. From Eq.(11) one finds:

$$f \;=\; \frac{\langle J/\psi\rangle}{N_{J/\psi}^{tot}}\;\left[(N_O/2)^2 + N_H\right] \;=\; g_c^2\;\left[(N_O/2)^2 + N_{J/\psi}^{tot}\right]\;. \tag{12}$$

This coincides with Eq.(5) at small values of $g_c N_O$.

The above calculations are restricted to a fixed number of participants $N_p = 400$. Let us check how the model works at smaller values of N_p. Using the experimental values of J/ψ multiplicities and assuming that the system volume V scales linearly with N_p we repeat the above calculations (the exact charm conservation becomes more and more important at smaller values of N_p) for $N_p = 100 \div 400$. The results are presented in Tables **A** and **B**, with the sets of the chemical freeze-out parameters **A** and **B**, respectively.

Table **A**

N_p	$\langle J/\psi\rangle \cdot 10^4$ Experim.	$N_{J/\psi}^{tot} \cdot 10^4$	N_O	g_c	$N_{c\bar{c}}^{dir}$	
					Stat.	Poisson
100	2.5	0.56	0.26	2.1	0.074	0.072
200	3.0	1.1	0.52	1.6	0.17	0.16
300	3.5	1.7	0.79	1.4	0.28	0.26
400	4.0	2.2	1.05	1.3	0.40	0.36

Table **B**

N_p	$\langle J/\psi\rangle \cdot 10^4$ Experim.	$N_{J/\psi}^{tot} \cdot 10^4$	N_O	g_c	$N_{c\bar{c}}^{dir}$	
					Stat.	Poisson
100	2.5	1.1	0.39	1.5	0.082	0.079
200	3.0	2.2	0.77	1.2	0.19	0.17
300	3.5	3.3	1.16	1.0	0.31	0.28
400	4.0	4.4	1.55	0.95	0.44	0.39

From results presented in Tables **A** and **B** we find that $N_{c\bar{c}}^{dir}$ scales as N_p^{α} with $\alpha \cong 1.20$ for the statistical probability distribution of the number of $c\bar{c}$ pairs (both for the sets **A** (8) and **B** (9)) and $\alpha \cong 1.15$ for the Poisson probability distribution (both for the sets **A** (8) and **B** (9)).

In conclusion, the statistical coalescence model with an exact charm conservation is formulated. The c.e. suppression effects are important for the thermal open charm yield. They become crucial when the number of participants N_p decreases. Using the J/ψ multiplicity data in Pb+Pb collisions at 158 A·GeV the considered model permits to calculate the open charm yield: $N_{c\bar{c}}^{dir} \cong 0.4$ in central collisions. The model predicts also its N_p dependence, $N_{c\bar{c}}^{dir} \sim N_p^{1.2}$, and the yields of individual open charm states. This is a prediction of the statistical coalescence model (the open charm yield has not been measured in Pb+Pb) and simultaneously a test (the measurements of the open charm are planned).

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